Resolution and Astrometric Accuracy

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We briefly consider how the distribution of baseline length effects both the beam FWHM and the astrometric accuracy.

Beam FWHM

Since the beam is symmetric through the origin slices through the beam only depend on the even powers of the angular displacement from the peak x. For most good uv coverages from the peak of the to the half power point the profile can be considered close to quadratic. In this case the FWHM

$$\theta_{res} \approx \sqrt{\frac{2}{d^2B(x,y)/dx^2}}$$

The second derivative through the beam equals

$$d^2B(x,y)/dx^2 = \frac{\int\int u^2 S(u,v)dudv}{\int\int S(u,v)dudv}$$

where S(u,v) is the u,v sampling function, consisting of delta functions at sampled uv points, hence

$$d^2B(x,y)/dx^2 = (1/N)\sum_i {u_i}^2$$

Where the sum i is over all uv points. For the case that the beam has the same resolution NS as EW then

$$d^2B(x,y)/dx^2 = d^2B(x,y)/dy^2 = (1/N)\sum_i {v_i}^2$$

or

$$d^2B(x,y)/dx^2 = (0.5/N)\sum_i {r_i}^2$$

Where r_i is the distance from the centre of the uv plane for the i'th uv point. Substituting into the first equation

$$\theta_{res} \approx \frac{1}{\sqrt{(1/N)\sum_i {r_i}^2}} = \frac{1}{\sqrt{(2/N)\sum_i {u_i}^2}}$$

Hence it follows that the beam FWHM of the natural weighted beam is inversely proportional to the mean square baseline length.

Accuracy of Phase Gradient Determination

Consider an unresolved source whose position we wish to determine. For simplicity consider that the position is unknown in RA but known in declination. [be nice to generalise this to 2D position finding where two gradients must be found by fitting the data with $\phi = ud\phi/du + vd\phi/dv$ - I don't think it will effect the argument below].

The phase gradient with respect to u is directly proportional to the distance between the correlated RA position and its actual position, therefore the accuracy or the RA position determination depends on the accuracy that the phase gradient can be measured.

Assume that object is unresolved and errors on phase are dominated by additive noise so that the phase noise on each measurement is constant, independent of uv distance [in fact even in the higher SNR limit when atmospheric phase noise dominates its going to be a weak function of uv distance for sizes of array we are talking about - but again would be nice to check]. Note also in the very low SNR case when the distribution of phase noise is far from gaussian if we can assume the error on position is much less than a beam the estimation of phase gradient with u reduces to the estimate of the gradient of the imaginary visibility with respect to u, and additive noise contributes gaussian errors on the imaginary part even at very low SNR). The argument below can then be followed assuming we are estimating gradients on imaginary part and not phase.

Given the assumption of baseline independent phase noise then each uv point i has an uncertainty in its measurement of phase gradient with respect to u of

$$\sigma_{gi} = \sigma / u_i$$

The most accurate estimate of the phase gradient comes from a weighted mean of all the individual measurements where the weighting factor for the i'th visibility is $w_i = 1/{\sigma_{gi}}^2$. The error on this weighted mean gradient obeys (Bevington 1992)

$${\sigma_o}^2 = 1/\sum (1/\sigma_{gi})^2$$

substituting for σ_{gi}

$$\sigma_o = \sigma / \sqrt{\sum {u_i}^2}$$

or

$$\sigma_o = \frac{\sigma/\sqrt{N}}{\sqrt{(1/N)\sum {u_i}^2)}}$$

where N is the number of uv points. So for two arrays with the same number of uv points the astrometric accuracy depends on the mean square baseline length (and the achieved SNR). Since the natural beam FWHMalso depends on the same quantity then to reasonable accuracy the astrometric error is proportional to the beam FWHM. Therefore two arrays with the same FWHM have approximately the same astrometric accuracy (given the assumptions stated in the above argument).